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Effect of Echo Canceler on Common-Channel Interoffice Signaling Continuity Check

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Annoying echoes generated by impedance mismatches can occur in long-distance telephone networks. An echo canceler controls echo by synthesizing an echo replica and subtracting it from the actual echo on the return path. For the echo canceler to work properly, it must be able to distinguish between desired signals and echoes. One instance where this can be difficult is during the common-channel interoffice signaling (CCIS) continuity check between four-wire switching offices. On CCIS trunks the voice and the signaling are routed separately. To ensure a satisfactory transmission path, a voice path continuity check is conducted before call setup. The check is performed on a loop basis by sending a check tone and looping it back at the distant office. Since the echo canceler adaptive filter memorizes information from the last previous call, it may partially cancel the looped-back tone. In this paper we study the effect of the echo canceler adaptive memory on the CCIS continuity check. The analytical and experimental results indicate that the occurrence of continuity check failure caused by the presence of an active echo canceler is so infrequent and insignificant compared to the existing statistics of all other pertinent failure mechanisms. Thus, disabling the echo canceler during the CCIS continuity check is unnecessary.

I. INTRODUCTION

An echo may be produced in a transmission system whenever there is an impedance mismatch. This impedance discontinuity can cause a significant portion of the transmitted signal energy to be reflected toward the signal source over an echo path. Noticeable echoes constitute one of the most serious forms of impairment in telephone channels. Its subjective annoyance increases with both echo amplitude and propagation delay. The increasing use of satellite circuits for domestic

and international calls has made the control of echoes even more important. The traditional approach of echo control using via net loss or echo suppressors is not acceptable for full-hop satellite circuits, i.e., circuits carrying both directions of transmission via satellite.[1,2] A better approach is to use echo cancelers that control echo by synthesizing a replica of the echo and subtracting it from the returned signal.[2,3] The realization of the single-chip integrated echo canceler has made its widespread deployment appealing.[4]

Figure 1 models the essential elements of the echo path to show how echo cancelers work. When there is far-end speech $x(t)$ but no near-end speech $v(t)$, the internal registers of the echo canceler adaptively update the estimate of the echo path impulse response $h(t)$ to form an echo replica $\hat{y}(t)$ that is subtracted from the real echo $y(t)$. When the near-end speech is detected in the presence of the far-end speech (double talk) the speech detector inhibits further updating, but the echo canceler still tries to cancel the echo contained in $y(t)$ by using the most recent estimate of the echo path impulse response. This property of continued echo canceling during double talk is a nice feature of the echo canceler. However, its effect on the common-channel interoffice signaling (CCIS) continuity check between four-wire switching offices has caused some concern.[5]

Common-channel interoffice signaling is a system for exchanging information between processor-equipped switching systems over a network of signaling links. All signaling data for call setup and take-down, as well as network management signals, are exchanged by these systems over the signaling links instead of being sent over the voice path. Thus, a continuity check for voice path assurance (VPA) is conducted whenever a CCIS trunk is selected to switch a call forward. This check not only ensures a satisfactory transmission path but also precludes billing for an otherwise undetectable faulty connection. Between four-wire switching offices the VPA check is performed on a loop basis by connecting a single-frequency transceiver at the originating office and looping the transmission pairs at the distant office. The check is considered successful when the VPA tone received at the originating office is within acceptable transmission and timing limits.

Figure 2 shows the VPA check for the satellite circuit. At the distant office the same notation $x(t)$ is used for the received and the transmitted signals because of the looping configuration. The echo canceler at the distant office will detect double talk during the VPA check. If the echo canceler is not disabled, it may partially cancel the check tone according to the last estimate of the echo path impulse response because of its continued canceling property during double talk. Therefore, the introduction of the echo canceler into the long-distance telephone network without proper disabling control will have some

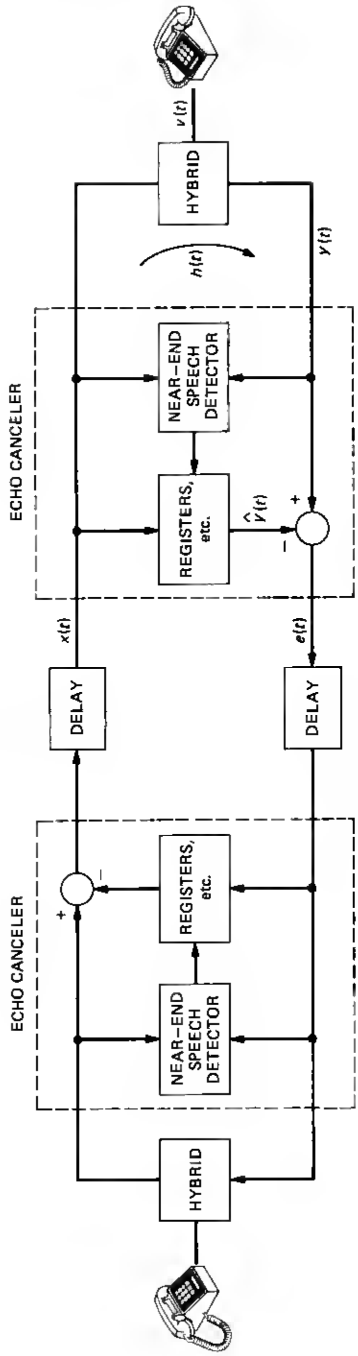

Fig. 1—Use of echo canceler to control echo.

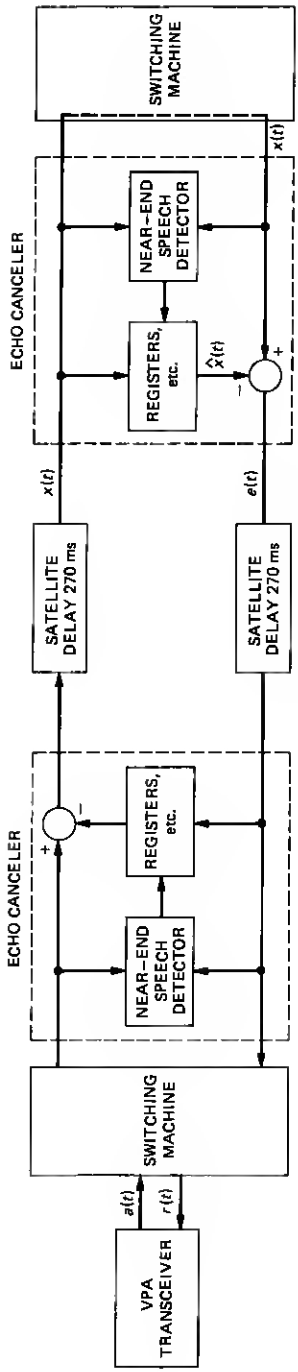

Fig. 2—Echo canceler and CCIS continuity check.

influence on the CCIS continuity check. Voice path assurance failures will lower the performance indices of the affected offices and generate additional maintenance activities. Unfortunately, since call processing information is not readily available, no satisfactory solution has been found such that the echo canceler can be disabled at the desired moment. The seriousness of the echo canceler effect on the VPA check is studied below by evaluating the probabilities of various possible events caused by VPA failures due to the use of the echo canceler. Section II describes the circuits and facilities in the VPA path and how they are modeled mathematically in the derivations given in the Appendices. Section III presents analytic and experimental results and discusses the impact of echo canceler on the CCIS continuity check.

II. MODELING OF VPA CHECK

In this section, the discussion and the derivation will be restricted to No. 4 ESS offices and satellite circuits because the results can be similarly derived for other facilities.

The VPA check for CCIS-equipped No. 4 ESS offices is a two-step test in a specified time interval. The first step is to detect at the proper receive level the transmitted VPA tone which is looped back at the distant office. The second step is to stop transmitting the VPA tone and detect its disappearance at the receiver. At present the specified time interval for completing the test is set anywhere between 2 to 3 s, depending on the specific office. At the beginning of a VPA check during call setup, the originating processor of the switching machine starts the timer and attaches a 2010-Hz transceiver to the selected satellite trunk concurrent with sending an address message through terrestrial CCIS links identifying the trunk to be looped back. The distant office, upon receipt of the message, connects the receive side of the trunk to the transmit side through a zero-loss loop. The originating office checks the level of the returning tone to verify that transmission loss is within acceptable limits. If the returning tone has acceptable level, the originating office stops sending the VPA tone and verifies that the receiver measures below a release level. A VPA failure is generated if for any reason the above two steps are not completed before the timer times out.

The specification of the transceiver levels is given in Table I. Frequent and periodic tests are performed in switching offices to make sure that the transceiver levels are within specification. Table I also gives the test requirements which are stricter than the specification.[6] Since the test only gives pass or fail indication, it is assumed that the various levels tested are uniformly distributed within the test limits. Thus, relative to each other, the transmit and the detect levels of the VPA tone are assumed to be uniformly distributed in $(-1, 1)$ dBm and

Table I—Common-channel interoffice signaling continuity check levels[1]

Test	Specification	Test Requirement
Transmit	$-12 + 1$ dBm0	$-12 \pm$ dBm0
Accept	$-18 + n \leq N$ dBm	$-18.6 + n \leq N$ dBm
Fail	$N \leq -22 + n$ dBm	$N \leq 21.4 + n$ dBm
Release	$N \leq -27 + n$ dBm	—

[1] Where N is the absolute power level of the VPA tone and n is the relative power at the transceiver with respect to the zero transmission level point.

(−9.4, −6.6) dBm, respectively. The release level in Table I is not modeled since a properly working echo canceler should not affect it.

In addition to the transceiver level variations, the VPA tone level is affected by the two-way satellite trunk loss, the check loop, and the echo canceler at the distant office. The satellite trunk is a class of intertoll trunks with 0 dB inserted connected loss which is specified to be normally distributed with a standard deviation of around 0.7 to 0.8 dB. The check loop is specified to have a loss of 0 ± 0.1 dB. This variation is small and will be ignored in the following study. The effect of the echo canceler requires detailed consideration.

In discrete-time notation, the near-end speech detector inhibits updating the echo canceler registers at time k if

$$y(k-l) > \tfrac{1}{2} \max_{0 \leq n \leq 127} x(k-l-n) \qquad \text{for} \quad 0 \leq l \leq 255, \tag{1}$$

where the notations are given in Fig. 1.[3] At an 8-kHz sampling rate, $0 \leq n \leq 127$ implies using a 16-ms tapped delay line adaptive digital filter to model the echo path, while $0 \leq l \leq 255$ means that once near-end speech is detected, the detector continues declaring its presence for the hangover time of 32 ms. The factor of one half is based on the assumption that there will be at least a 6-dB loss through the hybrid. During the continuity check, the VPA tone is looped back so the near-end speech detector finds that eq. (1) is satisfied and thus inhibits updating the registers. The echo canceler will partially cancel the VPA tone according to the register settings determined by the echo path return loss (EPRL) of the last call. Thus, the degree of cancellation is a random variable which is a function of the EPRL. The EPRL distribution of the whole telephone network is not known. The only available EPRL data, as shown in Fig. 3, were measured at the Pittsburgh Regional Center in the second half of 1976 during the field evaluation of domestic satellite.[7] These measurements are not inconsistent with previously reported echo path and intertoll trunk loss.[8] Although the loss distribution was derived from the estimated echo path frequency response as averages over 500 to 2500 Hz, it will be assumed to be the loss distribution for the VPA tone. The density function shown in

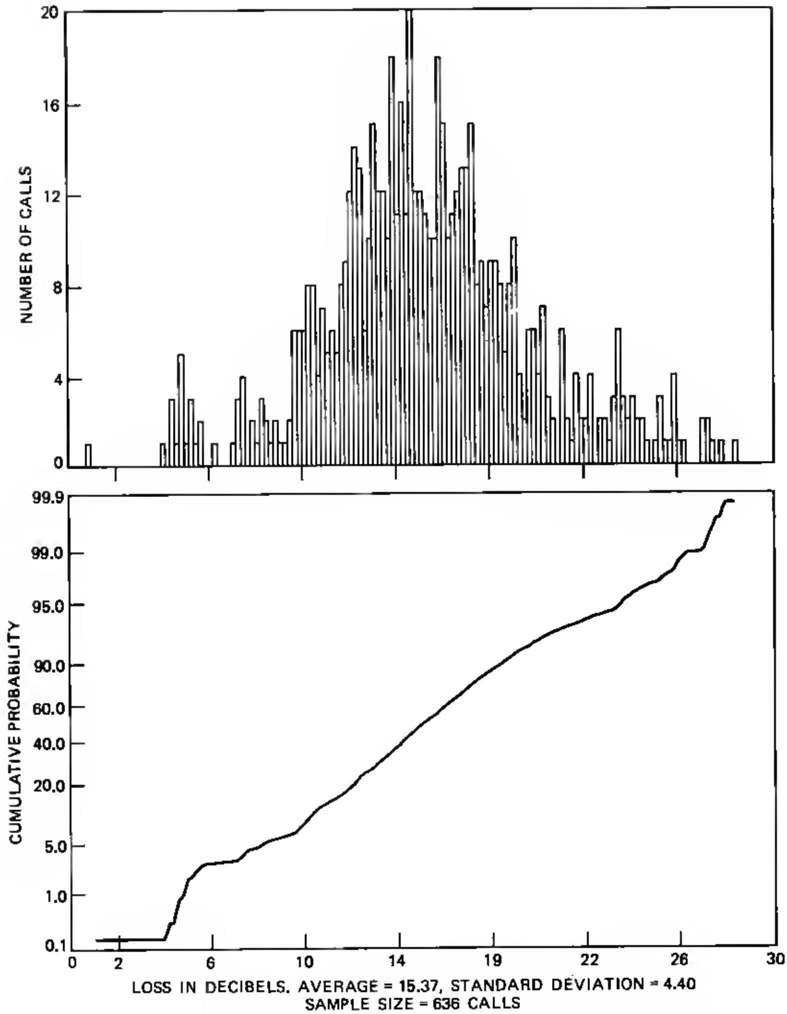

Fig. 3—Distribution of near-end echo path loss, 500 to 2500 Hz flat-weighted.

Fig. 3 is consistent with a normal distribution, except the small secondary peak at about 5 dB. The following derivation turns out to be independent of the loss distribution below 6 dB.

The above modeling of the VPA path can be used to predict VPA failures which may generate various kinds of maintenance activities affecting trunks and carrier groups. There are 12 trunks in each carrier group. If a trunk in a carrier group fails a VPA check, the call is reattempted on another trunk in a different carrier group, while the first carrier group is temporarily taken out-of-service. Several seconds

later, a second trunk in the locked-out carrier group is randomly selected and checked for continuity. If the second trunk also fails, a software carrier group alarm (SCGA) is generated for maintenance actions. If the second trunk passes the check, the temporary lockout on the first carrier group is released and the initially failed trunk is rechecked. If it fails again, a single trunk lockout (STL) is generated for maintenance. The probability of getting a VPA failure is derived in Appendix A. The probability of generating an STL is given in Appendix B. The probability of generating SCGA cannot be analyzed since the two failed trunks which share identical facilities from the switching offices to the satellite earth stations may be dependent. Thus, only the upper and the lower bounds of the probability of SCGA are obtained in Appendix C.

III. DISCUSSION OF RESULTS

Figures 4 to 7 show the results obtained in the appendices. During the VPA check, the estimated echo $\hat{x}(t)$ in Fig. 2 is also a single-frequency tone at 2010 Hz. Its amplitude and phase depend on the EPRL of the last call. For the worst-case, 6-dB EPRL, its effect on the looped back tone $x(t)$ varies from 3.5 dB (if the tones add constructively) to −6 dB (if the tones add destructively). With the EPRL distribution given in Fig. 3, the effect of the echo canceler on any VPA tone level follows the density function shown in Fig. 4. This density has a mean value slightly larger than zero; that is, the expected effect of the echo canceler is to increase the VPA tone level slightly. This is

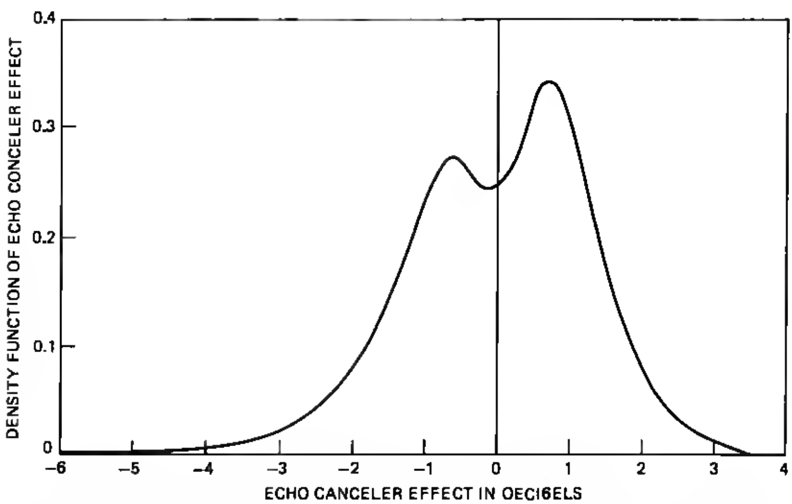

Fig. 4—Probability density function of the effect of the echo canceler on the VPA tone level.

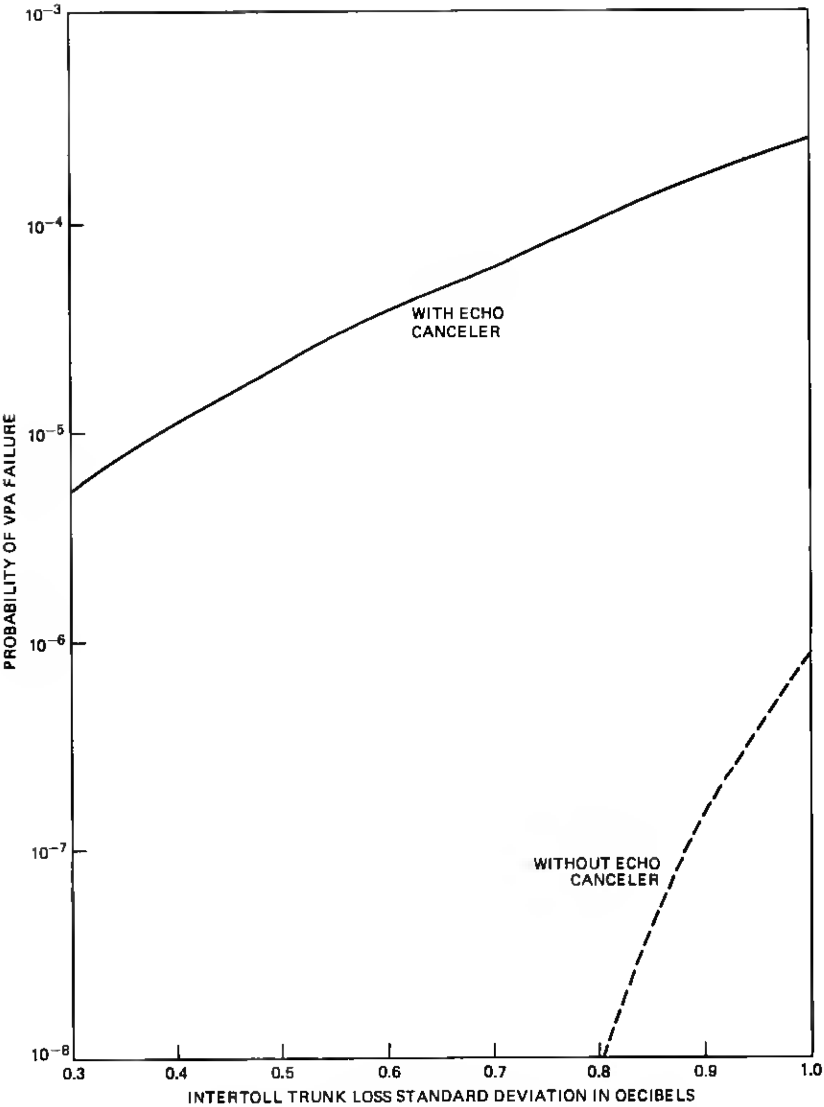

Fig. 5—Probability of VPA failure.

to be expected since the resultant VPA tone contains the powers of two sinusoids $x(t)$ and $\hat{x}(t)$ with random relative phases as opposed to the single sinusoid $x(t)$ without the echo canceler. However, the reducing effect on the VPA tone level has a greater impact because it can be as large as -6 dB. This is evident in Fig. 5, which shows that using echo canceler appreciably increases the probability of VPA failure. However,

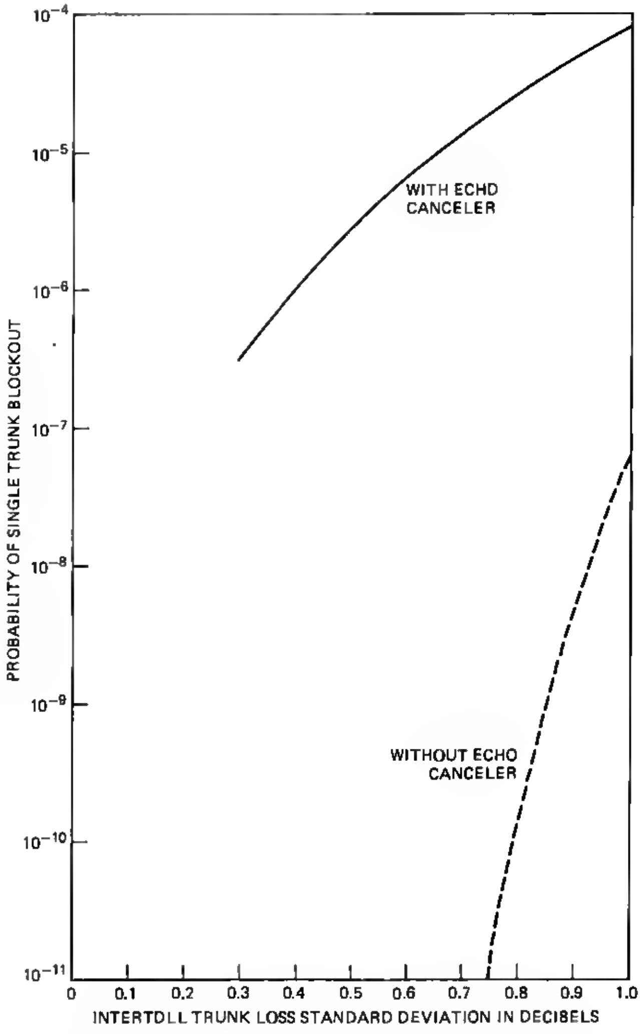

Fig. 6—Probability of single-trunk blockout.

the calculations assume that all circuits are working properly and without outside interferences. Voice path assurance failures can be generated by carrier glitch, fading, and many other causes. Limited sampling performed during the spring of 1980 in several No. 4 ESS offices not equipped with an echo canceler yielded VPA failure rates ranging from 0.5×10^{-5} to 2.7×10^{-4}, with a sample mean of 5.6×10^{-5}.[9] These are of the same order of magnitude as the calculated probability of VPA failure with echo canceler, assuming the satellite trunk standard deviation is between 0.7 to 0.8 dB. Therefore, if the

measured VPA failure rate for a group of trunks is 2×10^{-5}, it will be probably be around 4×10^{-5} after echo cancelers are installed on these trunks. The calculated probabilities of STB and SCGA as given in Figs. 6 and 7, respectively, also do not appear to be excessive, although there is no field statistics for comparison.

An experiment was performed on the full-hop satellite circuits between Atlanta, Georgia, and Cedar Knolls, New Jersey to see the effect of not disabling the echo canceler prior to the VPA check. The data indicated that there was no VPA failure for well over 100,000 calls.

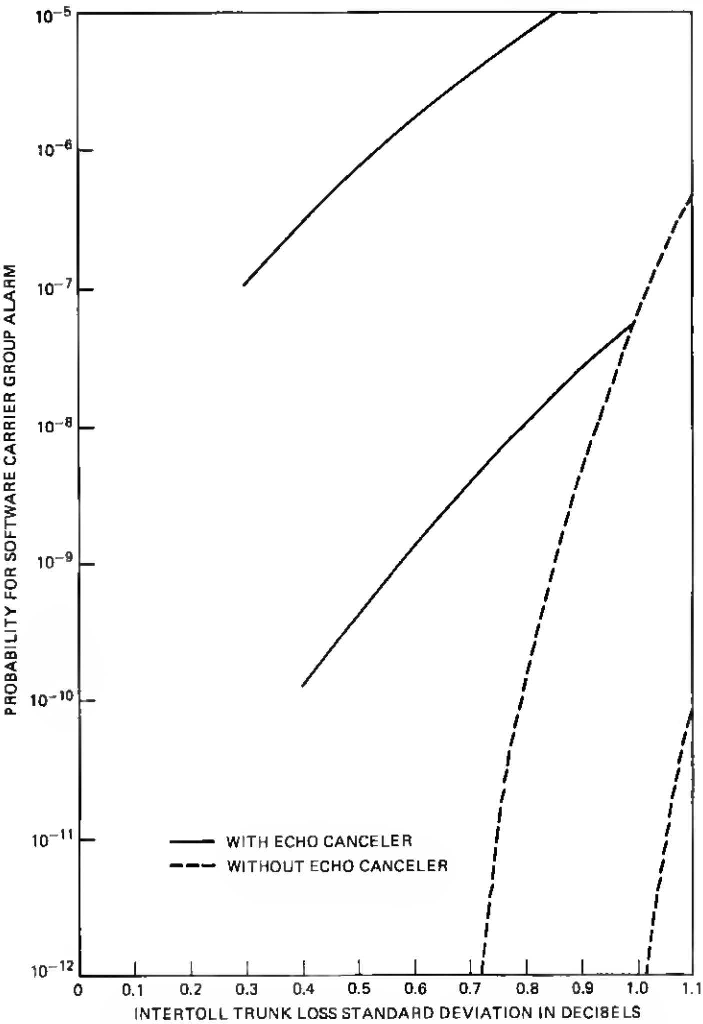

Fig. 7—Probability bounds for software carrier group alarm.

This result is better than the calculated probabilities and it can be easily explained. The calculated numbers are based on the specifications of the trunks and circuits. The standard deviation of the satellite trunks appears to be smaller than that of the intertoll trunks. Furthermore, limited measurements of the actual transceiver levels indicated that the transmit and the detect levels can be modeled as uniformly distributed in (−0.25, 0.25) dBm and (−8.75, −7.75) dBm, respectively.[10] All these factors help the performance of the VPA check.

In conclusion, the introduction of echo canceler to the satellite circuits may generate additional VPA failures. However, in present long distance networks, they are not significant compared to existing VPA failure statistics of the field. Thus, it does not appear necessary to disable the echo canceler prior to eacb CCIS continuity check.

APPENDIX A

The probability of VPA failure is derived in this appendix. Let l be the EPRL shown in Fig. 3; that is, l is normal with mean $\mu = 15.37$ and standard deviation $\sigma = 4.4$, except for $l < 6$ dB. By definition, for a single frequency tone,

$$
\begin{aligned}
l &\approx 10 \log \frac{\text{power of } x(t)}{\text{power of } y(t)} \\
&= 20 \log \frac{\text{amplitude of } x(t)}{\text{amplitude of } y(t)} \\
&\approx 20 \log \frac{\text{amplitude of } x(t)}{\text{amplitude of } \hat{y}(t)},
\end{aligned}
$$

where the last step follows for $l > 6$ dB. For the loopback configuration in Fig. 2 during the VPA check, then

$$l \approx 20 \log \frac{1}{b}, \qquad l > 6 \text{ dB}, \tag{1}$$

where

$$b = \frac{\text{amplitude of } \hat{x}(t)}{\text{amplitude of } x(t)}.$$

The derivation below assumes that eq. (1) is an exact equality. Since $l > 6$ dB, b is distributed in (0, ½). Let l_i, $i = 1, 2, \cdots$, denote the EPRL of the ith previous call using a specific echo canceler. The l_i's are identically distributed as l, and for most practical situations, independent random variables. Clearly b is a function of the l_i's. For instance, if $l_1 > 6$ dB, b is determined solely by l_1. If $l_1 < 6$ dB, the near-end speech detector would declare double talk and no register update would take place during the last call. Then b would be independent of

l_1 and become a function of only l_i, $i = 2, 3, \cdots$. Thus, the distribution function of b is given by

$$
\begin{aligned}
P\{b < c\} &= P\{10^{-l_1/20} < c\} + P\{l_1 < 6\}P\{10^{-l_2/20} < c\} \\
&\qquad + P\{l_1 < 6\}P\{l_2 < 6\}P\{10^{-l_3/20} < c\} + \cdots \\
&= (1 + (1-g) + (1-g)^2 + \cdots)P\{10^{-l/20} < c\} \\
&= \frac{1}{g} P\left\{ l > 20 \log \frac{1}{c} \right\} \\
&= \frac{1}{g} \int_{20 \log 1/c}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-(l-\mu)^2/2\sigma^2} dl, \qquad (2)
\end{aligned}
$$

where

$$
\begin{aligned}
g &= P\{l > 6\} \\
&= \int_6^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp[-(l-\mu)^2/2\sigma^2]\, dl.
\end{aligned}
$$

Differentiating eq. (2) gives the density function of b as

$$
f_b(b) = \frac{20}{\sqrt{2\pi}\sigma g \ln 10} \frac{1}{b} \exp[-(20 \log b + \mu)^2/2\sigma^2], \qquad 0 < b < \frac{1}{2}. \qquad (3)
$$

The transmitted VPA tone $a(t)$ can be written as

$$
a(t) = \sqrt{2}A \sin \omega t.
$$

The transmit level in dBm

$$
x_1 = 10 \log \frac{A^2}{10^{-3}}
$$

is uniformly distributed in $(-1, 1)$, as described in Section II. Since the satellite trunk loss in dB is normally distributed, it can be considered as gain for convenience. Assume the satellite trunk gains in the transmit and the receive directions are denoted by g_1 and g_2, respectively. Let

$$
x_2 = 10 \log g_1^2
$$

and

$$
x_3 = 10 \log g_2^2.
$$

The terms x_2 and x_3 are independent normal random variables with zero mean and standard deviation ρ. Thus, using the notation of Fig. 2,

$$x(t) = \sqrt{2}Ag_1 \sin \omega t,$$

$$e(t) = \sqrt{2}Ag_1 \sin \omega t - \sqrt{2}Ag_1 b \sin(\omega t + \phi)$$

$$= \sqrt{2}Ag_1 \sqrt{1 + b^2 - 2b \cos \phi} \cos \left[\omega t + \tan^{-1} \left(\frac{1 - b \cos \phi}{b \sin \phi} \right) \right],$$

where ϕ, the relative phase between the loopback signal $x(t)$ and the echo estimate $\hat{x}(t)$, is assumed to be uniformly distributed in $(0, 2\pi)$. The signal received by the transceiver

$$\gamma(t) = g_2 e(t)$$

has a power level in dBm of

$$\gamma = 10 \log \frac{A^2 g_1^2 g_2^2}{10^{-3}} (1 + b^2 - \cos \phi)$$

$$= x_1 + x_2 + x_3 + w, \tag{4}$$

where

$$w = 10 \log(1 + b^2 - 2b \cos \phi) \tag{5}$$

is the contribution of the echo canceler to the receive level of the VPA tone. Its effect can be evaluated by deriving the distribution of w. Since the density functions of b and ϕ are known, the density function of w can be shown to be

$$f_w(w) = \begin{cases} f_{w1}(w), & 10 \log 0.25 < w < 10 \log 0.75, \\ f_{w2}(w), & 10 \log 0.75 < w < 0, \\ f_{w3}(w), & 0 < w < 10 \log 2.25, \end{cases} \tag{6}$$

where

$$f_{w1}(w) = k_1 \int_{1.25-q_3}^{1} \frac{q_3 q_5}{q_4 q_2} dv,$$

$$f_{w2}(w) = k_1 \int_{\sqrt{1-q_3}}^{1} \frac{q_3 q_5}{q_4 q_2} dv + k_1 \int_{\sqrt{1-q_3}}^{1.25-q_3} \frac{q_3 q_6}{q_4 q_1} dv,$$

$$f_{w3}(w) = k_1 \int_{-1}^{1.25-q_3} \frac{q_3 q_6}{q_4 q_1} dv,$$

$$k_1 = \frac{1}{\sqrt{2\pi}\pi\sigma g},$$

$$q_1 = v + \sqrt{v^2 - 1 + q_3},$$

$$q_2 = v - \sqrt{v^2 - 1 + q_3},$$

$$q_3 = 10^{w/10},$$

$$q_4 = \sqrt{(v^2 - 1 + q_3)(1 - v^2)},$$

$$q_5 = \exp[-(20 \log q_2 + \mu)^2 / 2\sigma^2], \tag{7}$$

and

$$q_6 = \exp[-(20 \log q_1 + \mu)^2/2\sigma^2].$$

The density function $f_w(w)$ is plotted in Fig. 4. Since the mean value of $1 + b^2 - 2b \cos \phi$ is slightly larger than one, the expected value of w is slightly larger than zero.

The receive level γ in eq. (4) is the sum of random variables with known density functions; therefore, its distribution is also known. The transceiver detector level u is assumed to be uniformly distributed in (−9.4, −6.6) dBm. Thus, the probability of VPA failure with the echo canceler can be derived as

$$\begin{aligned} P_{ve} &= P\{\gamma < u\} \\ &= \frac{k_1}{11.2} \int_{-\infty}^{0} \Bigg\{ \Bigg[\int_{10\log 0.25}^{10\log 0.75} \int_{z-w-9.4}^{z-w-6.6} q_7\, dx \int_{1.25-q_3}^{1} \frac{q_3 q_5}{q_4 q_2}\, dv\, dw \\ &\quad + \int_{10\log 0.75}^{0} \int_{z-w-9.4}^{z-w-6.6} q_7\, dx \left(\int_{\sqrt{1-q_3}}^{1} \frac{q_3 q_5}{q_4 q_2}\, dv + \int_{\sqrt{1-q_3}}^{1.25-q_3} \frac{q_3 q_6}{q_4 q_1}\, dv \right) dw \\ &\quad + \int_{0}^{10\log 2.25} \int_{z-w-9.4}^{z-w-6.6} q_7\, dx \int_{1}^{1.25-q_3} \frac{q_3 q_6}{q_4 q_1}\, dv\, dw \Bigg\}\, dz. \end{aligned} \tag{8}$$

The probability of VPA failure without the echo canceler can be calculated as

$$P_v = P\{x_1 + x_2 + x_3 < u\} = \frac{1}{11.2} \int_{-\infty}^{0} \int_{z-9.4}^{z-6.6} q_7\, dx\, dz, \tag{9}$$

where

$$q_7 = \operatorname{erf}\left(\frac{x+1}{2\rho}\right) - \operatorname{erf}\left(\frac{x-1}{2\rho}\right)$$

and

$$\operatorname{erf}(x) = \frac{2}{\sqrt{2\pi}} \int_0^{\sqrt{2x}} e^{-y^2/2}\, dy$$

is the error function. Equations (8) and (9) are used to plot Fig. 5.

APPENDIX B

The probability of STL is derived in this Appendix. An STL is

generated on a trunk if the trunk fails a VPA check, if another trunk in the same carrier group passes a subsequent VPA check, and if the first trunk fails a VPA recheck. For simplicity, the second trunk is assumed to be independent of the first trunk. The probability that a trunk fails two consecutive VPA checks will be derived first. It is assumed that the small probability of selecting the same transceiver for the two VPA checks is negligible.

A VPA check on a trunk fails if $x_1 + x_2 + x_3 + w < u$. The values of x_2, x_3, and w remains unchanged for a VPA recheck on the same trunk. Thus, the probability of interest is $P\{x_2 + x_3 + w < u - x_1\}$. Let $z = u - x_1$, $s = x_2 + x_3$ and $y = x_2 + x_3 + w$. It can be shown that

$$f_z(z) = \begin{cases} \dfrac{1}{5.6}(10.4 + z), & -8.4 \geq z \geq -10.4, \\ \dfrac{2}{5.6}, & -7.6 \geq z \geq -8.4, \\ \dfrac{1}{5.6}(-z - 5.6), & -5.6 \geq z \geq -7.6, \end{cases}$$

$$f_s(s) = \frac{1}{2\sqrt{\pi}\rho}\exp(-s^2/4\rho^2), \tag{10}$$

and

$$f_y(y) = k_2 \int_{10\log 0.25}^{10\log 0.75} \int_{1.25-q_3}^{1} \frac{q_3 q_8 q_5}{q_4 q_2}\, dv\, dw$$
$$+ k_2 \int_{10\log 0.75}^{0} \left\{ \int_{\sqrt{1-q_3}}^{1} \frac{q_3 q_8 q_5}{q_4 q_2}\, dv + \int_{\sqrt{1-q_3}}^{1.25-q_3} \frac{q_3 q_8 q_6}{q_4 q_1}\, dv \right\}$$
$$+ k_2 \int_{0}^{10\log 2.25} \int_{-1}^{1.25-q_3} \frac{q_3 q_8 q_6}{q_4 q_1}\, dv\, dw,$$

where

$$k_2 = \frac{1}{2\sqrt{2}\pi^2 \sigma\rho g}$$

and

$$q_8 = \exp[-(y - w)^2/4\rho^2].$$

For a given value of y, a VPA failure is generated with the probability

$$P\{y < z\} = \int_y^\infty f_z(z)\,dz$$

$$= \begin{cases} 1, & y \le -10.4, \\ -\dfrac{1}{5.6}\left(48.48 + 10.4y + \dfrac{y^2}{2}\right), & -8.4 \ge y \ge -10.4, \\ -\dfrac{1}{5.6}(13.2 + 2y), & -7.6 \ge y \ge -8.4, \\ \dfrac{1}{5.6}\left(15.68 + 5.6y + \dfrac{y^2}{2}\right), & -5.6 \ge y \ge -7.6, \\ 0, & y \ge -5.6. \end{cases}$$

Therefore, the probability of two consecutive VPA failures on a trunk with echo canceler is

$$\begin{aligned} P_{2e} &= \int_{-\infty}^{\infty} P\{y < z\}P\{y < z\} f_y(y)\,dy \\ &= \int_{-\infty}^{-10.4} f_y(y)\,dy + \frac{1}{5.6^2}\int_{-10.4}^{-8.4}\left(48.48 + 10.4y + \frac{y^2}{2}\right)^2 \\ &\quad \times f_y(y)\,dy + \frac{1}{5.6^2}\int_{-8.4}^{-7.6}(13.2 + 2y)^2 f_y(y)\,dy \\ &\quad + \frac{1}{5.6^2}\int_{-7.6}^{-5.6}\left(15.68 + 5.6y + \frac{y^2}{2}\right)^2 f_y(y)\,dy. \end{aligned} \tag{11}$$

The probability of STL with echo canceler is then

$$P_{te} = P_{2e}\cdot(1 - P_{ve}). \tag{12}$$

Without echo canceler the probability of two consecutive VPA failures on a trunk, P_2, is obtained by replacing $f_y(y)$ in eq. (11) by $f_s(y)$ in (10). The probability of STL without echo canceler is then

$$P_t = P_2\cdot(1 - P_v). \tag{13}$$

Equations (12) and (13) are used to plot Fig. 6.

APPENDIX C

The probability of SCGA is studied in this appendix. An SCGA is generated if two trunks in the same carrier group fail successive VPA checks. If the two trunks are independent, the probability of SCGA is simply the square of the probability of VPA failure. This is plotted as

lower bounds in Fig. 7. Since the two trunks share identical carrier facilities from the switching offices to the earth stations, they may be dependent in some unknown way. A pessimistic estimate of the probability of SCGA is to assume that x_2 and x_3 remain the same for the two trunks selected. Thus, the probability of itnerest is $P\{x_1 + w - u < -(x_2 + x_3)\}$.

Let $t = x_1 + w$ and $h = t - u$. Then,

$$f_t(t) = \begin{cases} f_{t1}(t), & 10 \log 2.25 - 1 < t < 10 \log 2.25 + 1, \\ f_{t2}(t), & 1 < t < 10 \log 2.25 - 1, \\ f_{t3}(t), & 10 \log 0.75 + 1 < t < 1, \\ f_{t4}(t), & -1 < t < 10 \log 0.75 + 1, \\ f_{t5}(t), & 10 \log 0.75 - 1 < t < -1, \\ f_{t6}(t), & 10 \log 0.25 + 1 < t < 10 \log 0.75 - 1, \\ f_{t7}(t), & 10 \log 0.25 - 1 < t < 10 \log 0.25 + 1, \end{cases}$$

where

$$f_{t1}(t) = \frac{1}{2} \int_{t-1}^{10 \log 0.25} f_{w3}(w)\, dw,$$

$$f_{t2}(t) = \frac{1}{2} \int_{t-1}^{t+1} f_{w3}(w)\, dw,$$

$$f_{t3}(t) = \frac{1}{2} \int_{0}^{t+1} f_{w3}(w)\, dw + \frac{1}{2} \int_{t-1}^{0} f_{w2}(w)\, dw,$$

$$f_{t4}(t) = \frac{1}{2} \int_{0}^{t+1} f_{w3}(w)\, dw + \frac{1}{2} \int_{10 \log 0.75}^{0} f_{w2}(w)\, dw + \frac{1}{2} \int_{t-1}^{10 \log 0.75} f_{w1}(w)\, dw,$$

$$f_{t5}(t) = \int_{10 \log 0.75}^{t+1} f_{w2}(w)\, dw + \frac{1}{2} \int_{t-1}^{10 \log 0.75} f_{w1}(w)\, dw,$$

$$f_{t6}(t) = \frac{1}{2} \int_{t-1}^{t+1} f_{w1}(w)\, dw,$$

and

$$f_{t7}(t) = \frac{1}{2} \int_{10 \log 0.25}^{t+1} f_{w1}(w)\, dw.$$

The density function of h is given by

$$
f_h(h) = \begin{cases}
f_{h1}(h), & 10\log 2.25 + 8.4 < h < 10\log 2.25 + 10.4, \\
f_{h2}(h), & 10\log 2.25 + 7.6 < h < 10\log 2.25 + 8.4, \\
f_{h3}(h), & 10.4 < h < 10\log 2.25 + 7.6, \\
f_{h4}(h), & 10\log 0.75 + 10.4 < h < 10.4, \\
f_{h5}(h), & 10\log 0.75 + 5.6 < h < 10\log 0.75 + 10.4, \\
f_{h6}(h), & 8.4 < h < 10\log 2.25 + 5.6, \\
f_{h7}(h), & 7.6 < h < 8.4, \\
f_{h8}(h), & 10\log 0.75 + 8.4 < h < 7.6, \\
f_{h9}(h), & 10\log 0.75 + 7.6 < h < 10\log 0.75 + 8.4, \\
f_{h10}(h), & 5.6 < h < 10\log 0.75 + 7.6, \\
f_{h11}(h), & 10\log 0.25 + 10.4 < h < 5.6, \\
f_{h12}(h), & 10\log 0.75 + 5.6 < h < 10\log 0.25 + 10.4, \\
f_{h13}(h), & 10\log 0.25 + 8.4 < h < 10\log 0.75 + 5.6, \\
f_{h14}(h), & 10\log 0.25 + 7.6 < h < 10\log 0.25 + 8.4, \\
f_{h15}(h), & 10\log 0.25 + 5.6 < h < 10\log 0.25 + 7.6,
\end{cases}
$$

where

$$f_{h1}(h) = \frac{1}{2.8}\int_{h-9.4}^{10\log 2.25+1} f_{t1}(t)\,dt,$$

$$f_{h2}(h) = \frac{1}{2.8}\int_{10\log 2.25-1}^{10\log 2.25+1} f_{t1}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{10\log 2.25-1} f_{t2}(t)\,dt,$$

$$f_{h3}(h) = \frac{1}{2.8}\int_{10\log 2.25-1}^{h-6.6} f_{t1}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{10\log 2.25-1} f_{t2}(t)\,dt,$$

$$f_{h4}(h) = \frac{1}{2.8}\int_{10\log 2.25-1}^{h-6.6} f_{t1}(t)\,dt + \frac{1}{2.8}\int_{1}^{10\log 2.25-1} f_{t2}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{1} f_{t3}(t)\,dt,$$

$$f_{h5}(h) = \frac{1}{2.8}\int_{10\log 2.25-1}^{h-6.6} f_{t1}(t)\,dt + \frac{1}{2.8}\int_{1}^{10\log 2.25-1} f_{t2}(t)\,dt + \frac{1}{2.8}\int_{10\log 0.75+1}^{1} f_{t3}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{10\log 0.75+1} f_{t4}(t)\,dt,$$

$$f_{h6}(h) = \frac{1}{2.8}\int_{1}^{h-6.6} f_{t2}(t)\,dt + \frac{1}{2.8}\int_{10\log 0.75+1}^{1} f_{t3}(t)\,dt$$
$$+ \frac{1}{2.8}\int_{h-9.4}^{10\log 0.75+1} f_{t4}(t)\,dt,$$
$$f_{h7}(h) = \frac{1}{2.8}\int_{1}^{h-6.6} f_{t2}(t)\,dt + \frac{1}{2.8}\int_{10\log 0.75+1}^{1} f_{t3}(t)\,dt$$
$$+ \frac{1}{2.8}\int_{-1}^{10\log 0.75+1} f_{t4}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{-1} f_{t5}(t)\,dt,$$
$$f_{h8}(h) = \frac{1}{2.8}\int_{10\log 0.75+1}^{h-6.6} f_{t3}(t)\,dt + \frac{1}{2.8}\int_{-1}^{10\log 0.75+1} f_{t4}(t)\,dt$$
$$+ \frac{1}{2.8}\int_{h-9.4}^{-1} f_{t5}(t)\,dt,$$
$$f_{h9}(h) = \frac{1}{2.8}\int_{10\log 0.75+1}^{h-6.6} f_{t3}(t)\,dt + \frac{1}{2.8}\int_{10\log 0.75-1}^{-1} f_{t5}(t)\,dt$$
$$+ \frac{1}{2.8}\int_{-1}^{10\log 0.75+1} f_{t4}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{10\log 0.75-1} f_{t6}(t)\,dt,$$
$$f_{h10}(h) = \frac{1}{2.8}\int_{-1}^{h-6.6} f_{t4}(t)\,dt + \frac{1}{2.8}\int_{10\log 0.75-1}^{-1} f_{t5}(t)\,dt$$
$$+ \frac{1}{2.8}\int_{h-9.4}^{10\log 0.75-1} f_{t6}(t)\,dt,$$
$$f_{h11}(h) = = \frac{1}{2.8}\int_{10\log 0.75-1}^{h-6.6} f_{t5}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{10\log 0.75-1} f_{t6}(t)\,dt,$$
$$f_{h12}(h) = \frac{1}{2.8}\int_{10\log 0.75-1}^{h-6.6} f_{t5}(t)\,dt + \frac{1}{2.8}\int_{10\log 0.25+1}^{10\log 0.75-1} f_{t6}(t)\,dt$$
$$+ \frac{1}{2.8}\int_{h-9.4}^{10\log 0.25+1} f_{t7}(t)\,dt,$$
$$f_{h13}(h) = \frac{1}{2.8}\int_{10\log 0.25+1}^{h-6.6} f_{t6}(t)\,dt + \frac{1}{2.8}\int_{h-9.4}^{10\log 0.25+1} f_{t7}(t)\,dt$$
$$f_{h14}(h) = \frac{1}{2.8}\int_{10\log 0.25+1}^{h-6.6} f_{t6}(t)\,dt + \frac{1}{2.8}\int_{10\log 0.25-1}^{10\log 0.25+1} f_{t7}(t)\,dt,$$
$$f_{h15}(h) = \frac{1}{2.8}\int_{10\log 0.25-1}^{h-6.6} f_{t7}(t)\,dt.$$

For a given value of s, a VPA failure is generated with the probability

$$
P\{h < -s\} = \int_{-\infty}^{-s} f_h(h)\, dh
$$

$$
= \begin{cases}
0, \quad \text{for} \quad -s < 10\log 0.25 + 5.6, \\[4pt]
\displaystyle\int_{10\log 0.25+5.6}^{-s} f_{h15}(h)\, dh, \\
\quad \text{for} \quad 10\log 0.25 + 5.6 < -s < 10\log 0.25 + 7.6, \\[4pt]
\displaystyle\int_{10\log 0.25+5.6}^{10\log 0.25+7.6} f_{h15}(h)\, dh + \int_{10\log 0.25+7.6}^{-s} f_{h14}(h)\, dh, \\
\quad \text{for} \quad 10\log 0.25 + 7.6 < -s < 10\log 0.25 + 8.4, \\[4pt]
\displaystyle\int_{10\log 0.25+5.6}^{10\log 0.25+7.6} f_{h15}(h)\, dh + \int_{10\log 0.25+7.6}^{10\log 0.25+8.4} f_{h14}(h)\, dh \\
\quad \displaystyle + \int_{10\log 0.25+8.4}^{-s} f_{h13}(h)\, dh, \\
\quad \text{for} \quad 10\log 0.25 + 8.4 < -s < 10\log 0.75 + 5.6, \\[4pt]
\displaystyle\int f_{h15} + \int f_{h14} + \int_{10\log 0.25+8.4}^{10\log 0.75+5.6} f_{h13}(h)\, dh \\
\quad \displaystyle + \int_{10\log 0.75+5.6}^{-s} f_{h12}(h)\, dh, \\
\quad \text{for} \quad 10\log 0.75 + 5.6 < -s < 10\log 0.25 + 10.4 \\
\qquad \vdots \\
\displaystyle\int f_{h15} + \int f_{h14} + \cdots + \int_{10\log 2.25+7.6}^{10\log 2.25+8.4} f_{h2}(h)\, dh \\
\quad \displaystyle + \int_{10\log 2.25+8.4}^{-s} f_{h1}(h)\, dh, \\
\quad \text{for} \quad 10\log 2.25 + 8.4 < -s < 10\log 2.25 + 10.4 \\[4pt]
1, \quad \text{for} \quad 10\log 2.25 + 10.4 < -s.
\end{cases}
$$

Let $p = -s$. The pessimistic estimate of the probability of SCGA with echo canceler is given by

$$\int_{-\infty}^{\infty} P\{h < p\} P\{h < p\} f_s(p)\, dp$$

$$= \int_{10\log 0.25+5.6}^{10\log 0.25+7.6} \left(\int_{10\log 0.25+5.6}^{p} f_{h15}(h)\, dh \right)^2 f_s(p)\, dp$$

$$+ \int_{10\log 0.25+7.6}^{10\log 0.25+8.4} \left(\int_{10\log 0.25+5.6}^{10\log 0.25+7.6} f_{h15}(h)\, dh \right.$$

$$\left. + \int_{10\log 0.25+7.6}^{p} f_{h14}(h)\, dh \right)^2 f_s(p)\, dp$$

$$+ \cdots + \int_{10\log 2.25+8.4}^{10\log 2.25+10.4} \left(\int f_{h15} + \int f_{h14} + \cdots \right.$$

$$\left. + \int_{10\log 2.25+7.6}^{10\log 2.25+8.4} f_{h2}(h)\, dh + \int_{10\log 2.25+8.4}^{p} f_{h1}(h)\, dh \right)^2 f_s(p)\, dp$$

$$+ \int_{10\log 2.25+10.4}^{\infty} (5.6)^2 f_s(p)\, dp. \qquad (14)$$

Without echo canceler, the pessimistic estimate of the probability of SCGA is simply P_t, the probability of STL given in eq. (13). Equations (13) and (14) are used to plot the upper bounds of Fig. 7.

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